A relativistic quantum kinetic equation for nucleus-nucleus collisions

Botermans, Wim; Malfiet, Rudi

Published in:
Physics Letters B

DOI:
10.1016/0370-2693(88)90031-7

Publication date:
1988

A RELATIVISTIC QUANTUM KINETIC EQUATION FOR NUCLEUS–NUCLEUS COLLISIONS

Wim BOTERMANS and Rudi MALFLIET
Kernfysisch Versneller Instituut, Zernikelaan 25, NL-9747 AA Groningen, The Netherlands

Received 29 July 1988; revised manuscript received 5 October 1988

A relativistic quantum kinetic equation is derived corresponding to the non-equilibrium extension of the Dirac–Brueckner approach for nuclear matter. The equation is of the VUU-type with a self-consistent mean field and collision term.

From recent studies of equilibrium properties of nuclear matter and finite nuclei it has become clear that besides a microscopic quantum many-body approach like the Brueckner–Bethe–Goldstone method, also relativity is an important ingredient to describe the observables correctly. This observation leads one to find a relativistic quantum kinetic equation, i.e. a non-equilibrium formulation of the aforementioned equilibrium theory. The general cornerstones for such a formulation can be found in the work of Martin and Schwinger [1], Kadanoff and Baym [2] (non-relativistic kinetic theory) and de Groot et al. [3] (relativistic kinetic theory).

In this paper we present the derivation of a relativistic kinetic equation, starting from a one-boson-exchange (OBE) nucleon–nucleon interaction lagrangian. The derivation itself is very much along the same lines as we pursued in the non-relativistic case [4]. In both cases the crucial step is a specific truncation of the hierarchy of equations of motion. The OBE-interaction is the result of the exchange of scalar (σ, δ), vector (ω, ρ) and pseudo-vector (π, η) mesons. From the effective lagrangian density [5] we obtain the following Euler–Lagrange equations:

$$(\mathrm{i}\gamma_\mu\partial^\mu - M)\psi = [\lambda_\mathrm{s}\phi_\mathrm{s} + \mathrm{i}(\lambda_\mathrm{pv}/2M)\gamma_5\gamma_\mu(\partial^\mu\phi_\mathrm{pv}) + \lambda_\mathrm{v}\gamma_\mu\phi_\mathrm{v}^\mu + (\lambda_\mathrm{v}'/2M)\sigma_{\mu\nu}(\partial^\mu\phi_\mathrm{v}^\nu)]\psi, \tag{1}$$

$$(\Box + m_\mathrm{s}^2)\phi_\mathrm{s} = -\lambda_\mathrm{s}\bar{\psi}\psi, \quad (\Box + m_\mathrm{pv}^2)\phi_\mathrm{pv} = -\mathrm{i}(\lambda_\mathrm{pv}/2M)\partial_\mu(\bar{\psi}\gamma_5\gamma^\mu\psi), \tag{2,3}$$

$$\partial_\nu F^{\nu\mu} + m_\mathrm{v}^2\phi_\mathrm{v}^\mu = \lambda_\mathrm{v}\bar{\psi}\gamma^\mu\psi - (\lambda_\mathrm{v}'/2M)\partial_\nu(\bar{\psi}\sigma^{\nu\mu}\psi) \tag{4}$$

with

$$F^{\mu\nu} = \partial^\mu a^\nu_\mathrm{v} - \partial^\nu\phi_\mathrm{v}^\mu. \tag{5}$$

In these equations we see that meson fields and nucleon fields are coupled. However, for this particular form of the interaction lagrangian, there is a nice method, due to Wilets et al. [6], to eliminate the meson fields formally from the problem. One inverts the meson equations of motion (2)–(4) and substitute the solutions into the equation of motion for the nuclear field, eq. (1). Denoting by the label 1, 2, ..., a four-vector and a spinor component $(\boldsymbol{x}_1, t_1, \alpha_1)$, $(\boldsymbol{x}_2, t_2, \alpha_2)$ ... and summing and integrating over repeated indices, the result reads

$$S(1,1')\,\psi(1') = \langle 12|V|1''2''\rangle\,\bar{\psi}(2)\,\psi(2'')\,\psi(1''), \tag{6}$$

where

$$\begin{aligned}\langle 12|V|1'2'\rangle = {} & -\Gamma^\mathrm{s}(1,1',\zeta)\,g^\mathrm{s}(\zeta,\zeta')\,\Gamma^\mathrm{s}(2,2',\zeta') + [\partial_\xi^\mu\Gamma_\mu^\mathrm{pv}(1,1',\zeta)]\,g^\mathrm{pv}(\zeta,\zeta')\,[\partial_{\zeta'}^\nu\Gamma_\nu^\mathrm{pv}(2,2',\zeta')] \\ & - [\Gamma_\mu^\mathrm{v}(1,1',\zeta) - \partial_\xi^\rho\Gamma_{\rho\mu}^{\mathrm{v}'}(1,1',\zeta)]\,g_\mathrm{v}^{\mu\nu}(\zeta,\zeta')\,[\Gamma_\nu^\mathrm{v}(2,2',\zeta') - \partial_{\zeta'}^\tau\Gamma_{\tau\nu}^{\mathrm{v}'}(2,2',\zeta')],\end{aligned} \tag{7}$$

and furthermore we used

0370-2693/88/$ 03.50

$$S(1,1') = (i\gamma_\mu \partial^\mu - M)_{\alpha_1\alpha_1'} \delta^4(x_1 - x_1'), \quad (\Box_\xi + m_j^2) g^{s,pv}(\zeta,\zeta') = \delta(\zeta - \zeta'), \tag{8,9}$$

$$(g^{\mu\nu}\Box_\xi - \partial^\mu\partial^\nu + g^{\mu\nu}m_v^2) g^v_{\nu\rho}(\zeta,\zeta') = \partial^\mu_\rho \delta(\zeta - \zeta'), \quad \Gamma^j(1,1',\zeta) = \Gamma^j \delta^4(x_1 - x_1')\delta^4(x_1 - x_\zeta), \tag{10,11}$$

$$\Gamma^s = \lambda_s, \quad \Gamma^v_\mu = \lambda_v \gamma_\mu, \quad \Gamma^{v'}_{\mu\nu} = (\lambda_v'/2M)\sigma_{\mu\nu}, \quad \Gamma^{pv}_\mu = i(\lambda_{pv}/2M)\gamma_5\gamma_\mu.$$

Since in the V-matrix element (7) the Γ^j are vertices and the g^j are meson Green's functions this matrix element is exactly the OBE interaction. While in actual calculations of the OBE-interaction retardation is usually neglected, here it is still taken into account. Form factors are not included here but can be inserted in a straightforward way. The equation of motion of the nucleon field equation (6) is now expressed only in terms of other nucleon fields and it is this property that enables us to construct a hierarchy of nucleon Green's functions.

Let us first define the anti-causal respectively causal N-particle Green's function

$$G^{a,c}(1...N, 1'...N') = (1/i)^N \langle \phi_0 | T^{a,c}(\psi(1)...\psi(N)\bar\psi(N')...\bar\psi(1')) | \phi_0 \rangle \tag{12}$$

and their correlation functions

$$G^>(1...N, 1'...N') = (1/i)^N \langle \phi_0 | \psi(1)...\psi(N)\bar\psi(N')...\bar\psi(1') | \phi_0 \rangle,$$

$$G^<(1...N, 1'...N') = (i)^N \langle \phi_0 | \bar\psi(1')...\bar\psi(N')\psi(N)...\psi(1) | \phi_0 \rangle, \tag{13,14}$$

with $T^{a,c}$ the anti-chronological respectively chronological time ordering operator and $|\phi_0\rangle$ the asymptotic initial state of the heavy-ion reaction. Note that we work in the Heisenberg picture where the time-dependent field-operators describe the time development of the system. In this case the correlation function $G_1^<$ describes the particle distribution and thus is the appropriate quantity to express the kinetic equation in. Its equation of motion is most elegantly given in the real-time formalism developed by Keldysh [7] and Craig [8]. In this formalism one defines Green's functions $\underline{G}$ on a directed contour C which runs from time $-\infty$ to $+\infty$ on the upper branch of the real time axis and then from time $+\infty$ to $-\infty$ on the lower branch of it. Green's functions defined on contour C have the foregoing functions (12)–(14) as special cases. The N-particle contour Green's function is defined as

$$\underline{G}(1...N, 1'...N') = (1/i)^N \langle \phi_0 | P(\psi(1)...\psi(N)\bar\psi(N')...\bar\psi(1')) | \phi_0 \rangle. \tag{15}$$

The path-ordering operator P orders the field operators along contour C in such a way that field operators "later" on the contour are put to the left of "earlier" ones. The sign convention is similar to that of $T^{a,c}$. The special cases show up when we pin down the branches of the times involved. For instance if in the one-body Green's function $\underline{G}(1,1')$ we take both times on the upper branch we get the causal Green's function $G^c(1,1')$. The equations of motion of these very general Green's functions form a hierarchy, since the equation of motion of the N-particle Green's function is given in terms of the $(N-1)$-particle and $(N+1)$-particle Green's function. Formally it is similar to the so-called Martin–Schwinger hierarchy [1]. The first equation of it has the form [#1]

$$\underline{S}(1,1'')\,\underline{G}(1'',1') = \underline{\delta}(1-1') - i\langle 12|\underline{V}|1''2''\rangle\,\underline{G}(1''2'',1'2^+). \tag{16}$$

The underlining of the functions indicate that these functions are now defined on contour C and the repeated indices are now integrated over contour C. For the matrix elements of $\underline{S}$ and $\underline{V}$, the underlining indicates that in their definitions (8) and (11), the delta function $\delta^4(x_1 - x_1')$ is replaced by $\underline{\delta}^4(x_1 - x_1')$. We can formally rewrite (16) as a Dyson equation

$$\underline{S}(1,1'')\,\underline{G}('',1') = \underline{\delta}(1-1') + \underline{\Sigma}(1,1'')\,\underline{G}(1'',1'), \tag{17}$$

where the mass operator Σ is given by

[#1] The + sign in (16) indicates a point infinitesimal further on the contour. The delta function is defined as $\delta(1-1') = \delta(1-1')$ if both t_1 and t_1' are on the upper branch, $\delta(1-1') = -\delta(1-1')$ if both t_1 and t_1' are on the lower branch, $\delta(1-1') = 0$ in other cases.

$$\underline{\Sigma}(1,1') = -\mathrm{i}\langle 12|\underline{V}|1''2''\rangle\, \underline{G}(1''2'', 1'''2^+)\, \underline{G}^{-1}(1''', 1'). \tag{18}$$

From eq. (17) we see that if we find an expression for the mass operator we have solved the whole problem. Eq. (18) gives the relation of this mass operator to the two-particle Green's function. For the latter, one can obtain an approximate expression, directly from the Martin–Schwinger hierarchy. To that end we form a differential equation for two two-particle Green's function, linearise the equation by factorising the appearing higher-order Green's functions and solve the linear equation subject to a boundary condition [5,9]. This boundary condition we take to be Boltzmann's assumption of molecular chaos, since we believe that it is one of the basic elements of a Boltzmann-like kinetic equation. Upon substituting the approximate solution for the two-particle Green's function into equation (18), we find for the mass-operator

$$\underline{\Sigma}(1,1') = -\mathrm{i}(\langle 12|\underline{T}|1'2'\rangle - \langle 12|\underline{T}|2'1'\rangle)\, \underline{G}(2', 2^+), \tag{19}$$

where the generalized T-matrix is the solution of

$$\langle 12|\underline{T}|1'2'\rangle = \langle 12|\underline{V}|1'2'\rangle + \mathrm{i}\langle 12|\underline{V}|1''2''\rangle\, \underline{G}(1'', 1''')\, \underline{G}(2'', 2''')\langle 1'''2'''|\underline{T}|1'2'\rangle. \tag{20}$$

Eq. (19) gives the mass-operator in the so-called T-approximation [2]. Together with the Dyson equation (17) it solves the problem. This T-matrix equation, defined on the time-directed contour C, has again four components which describe the effective interaction as well as the collision dynamics. Though not apparently clear from expression (20), the intermediate states of the effective interaction respect the Pauli-principle in the correct way. This can be inferred from the explicit form of the equation for the effective interaction (eq. (45)), which follows directly from the expression (20). We remark that the Green's functions appearing in (17), (19) and (20) are the same, thus we deal with a self-consistent set of equations. Furthermore, it is a conserving approximation in the sense of Baym [10]. To obtain our kinetic equation we first introduce retarded (+) and advanced (−) functions by

$$F^{(\pm)}(1,1') = F^{\mathrm{c}}(1,1') - F^{>}(1,1') = F^{\lessgtr}(1,1') - F^{\mathrm{a}}(1,1'). \tag{21}$$

With these definitions we find from the Dyson equation (17) the equation of motion for $G^<$

$$S(1,1'')\, G^<(1'',1') = \Sigma^{(+)}(1,1'')\, G^<(1'',1') + \Sigma^<(1,1'')\, G^{(-)}(1'',1'). \tag{22}$$

From the Dyson equation (17) and the definition (21) the equation of motion of $G^{(\pm)}$ can be obtained also

$$S(1,1'')\, G^{(\pm)}(1'',1') = \delta(1-1') + \Sigma^{(\pm)}(1,1'')\, G^{(\pm)}(1'',1'). \tag{23}$$

Note that the time integration now runs from $-\infty$ to ∞, since we fixed the time branches. For details we refer to refs. [5,9]. Eqs. (22), (23) and their adjoints, will be the basic equations from now on. The kinetic equation is given by the subtraction of eq. (22) and its adjoint. When we separate in this kinetic equation the real and imaginary parts of $\Sigma^{(\pm)}$ and $G^{(\pm)}$, we get

$$\begin{aligned} &\mathrm{i}\partial_\mu \gamma^\mu G_1^< + \mathrm{i}G_1^< \gamma^\mu \overleftarrow{\partial}_\mu - (\mathrm{Re}\,\Sigma_1^{\{+\}} G_1^< - G_1^< \,\mathrm{Re}\,\Sigma_1^{\{+\}}) - (\Sigma_1^< \,\mathrm{Re}\, G_1^{\{+\}} - \mathrm{Re}\, G_1^{\{+\}} \Sigma_1^<) \\ &\quad = \tfrac{1}{2}\,[\Sigma_1^> G_1^< + G_1^< \Sigma_1^> - \Sigma_1^< G_1^> - G_1^> \Sigma_1^<], \end{aligned} \tag{24}$$

where [#2]

$$\Sigma_1^{\lessgtr} = -\mathrm{i}\,\mathrm{Tr}_{(2)}\, T_{12}^{\lessgtr}\, \mathscr{A}_{12} G_2^{\lessgtr}, \quad \Sigma_1^{\{\pm\}} = -\mathrm{i}\,\mathrm{Tr}_{(2)}\,[T_{12}^{\{\pm\}} \mathscr{A}_{12} G_2^< + T_{12}^< \mathscr{A}_{12} G_2^{\{\mp\}}] \tag{25}$$

and

$$T_{12}^{\{\pm\}} = V_{12} + \mathrm{i}V_{12} G_{12}^{\{\pm\}} T_{12}^{\{\pm\}}, \quad T_{12}^{\lessgtr} = \mathrm{i}T_{12}^{\{+\}} G_1^{\lessgtr} G_2^{\lessgtr} T_{12}^{\{-\}}. \tag{26}$$

[#2] In these expressions $\mathscr{A}_{12}$ is the anti-symmetrisation operator, defined as $\mathscr{A}_{12} = 1 - \mathscr{P}_{12}$, where $\mathscr{P}_{12}$ interchanges particle 1 and 2.

Together with the expressions for $\Sigma^{(\pm)}$ and $\Sigma^{\gtrless}$, which can be inferred from (19) and (20), eq. (24) constitute the kinetic equation. However, its non-local character makes it a highly unpractical equation. For this reason we switch to the Wigner representation. The Wigner representation $f_{\alpha_1\alpha_1'}(p, R)$ for a function $F_{\alpha_1\alpha_1'}(x_1, x_1')$ is defined by introducing new variables $R=\frac{1}{2}(x_1+x_1')$, $r=x_1-x_1'$ and Fourier transforming only the latter one

$$f_{\alpha_1\alpha_1'}(p, R)=\int \mathrm{d}^4 r \exp(\mathrm{i}pr) F_{\alpha_1\alpha_1'}(R+r/2, R-r/2). \tag{27}$$

Terms in which a product of two functions appears, like in eq. (24), we approximate by taking only the first-order contribution in the gradients [#3]

$$(f\otimes g)_{\alpha_1\alpha_1'}(p, R)\equiv f_{\alpha_1\alpha_1''}(p, R) g_{\alpha_1''\alpha_1'}(p, R)+\tfrac{1}{2}\mathrm{i}\{f_{\alpha_1\alpha_1''}(p, R), g_{\alpha_1''\alpha_1'}(p, R)\}, \tag{28}$$

based on the assumption of slowly varying disturbances [2].

We now change from spinor to spin-representation as follows. Here we restrict ourselves, in accordance with the standard Dirac–Brueckner theory [11], to positive-energy spinors only (α is the spinor index, r labels the spin z-component) [#4]

$$G^{<}_{\alpha\alpha'}(p, R)=\mathrm{i}\sum_{r,r'=1}^{2} \tilde{u}^{r}_{\alpha}(p, R)\, \bar{\tilde{u}}^{r'}_{\alpha'}(p, R) A(p, R) 2\theta(p_0^*) F^{rr'}(p, R), \tag{29}$$

$$G^{>}_{\alpha\alpha'}(p, R)=-\mathrm{i}\sum_{r,r'=1}^{2} \tilde{u}^{r}_{\alpha}(p, R)\, \bar{\tilde{u}}^{r'}_{\alpha'}(p, R) A(p, R)\, [\delta_{rr'}-2\theta(p_0^*) F^{rr'}(p, R)]. \tag{30}$$

The factor 2 is introduced to be consistent with the standard spin-representation [3]. Compared to that one, there are two differences.

Firstly, we use effective spinors denoted as $\tilde{u}$ rather than free ones in the decomposition (29), (30). Effective spinors play a crucial role in the Dirac–Brueckner theory of nuclear matter [11]. They are solutions of a Dirac equation with a self-energy contribution. For nuclear matter this self-energy, which is the real part of the mass operator, in general has the following Lorentz structure (the minus sign is conventional:

$$\mathrm{Re}\,\Sigma^{(+)}_{\alpha\alpha'}(p, R)=\mathrm{Re}\,\Sigma^{\mathrm{s}}(p, R) 1_{\alpha\alpha'}-\mathrm{Re}(\Sigma^{\mathrm{v}})_{\mu}(p, R)(\gamma^{\mu})_{\alpha\alpha'} \tag{31}$$

If we introduce the effective mass M_p^* and effective momentum p^* as

$$M_p^*=M+\mathrm{Re}\,\Sigma^{\mathrm{s}}(p, R), \quad p^{*\mu}=p^{\mu}+\mathrm{Re}(\Sigma^{\mathrm{v}})^{\mu}(p, R), \tag{32,33}$$

the Dirac equation we started with turns into a free one, with M_p^*, $p^{*\mu}$ instead of M, p^{μ}. Consequently, its solution, the effective spinor $\tilde{u}$, has the same formal appearance as a free spinor only with effective quantities M_p^*, $p^{*\mu}$.

The second deviation in expressions (29), (30) from the standard ones, is the introduction of the spectral function A, known from non-relativistic treatments [2]. Here its definition is

$$\begin{aligned} A(p, R)&=\frac{\mathrm{i}}{2}\sum_{r=1}^{2} \bar{\tilde{u}}^{r}(p, R)\,[G^{>}(p, R)-G^{<}(p, R)]\,\tilde{u}^{r}(p, R) \\ &=\frac{\mathrm{i}}{2}\sum_{r=1}^{2} \bar{\tilde{u}}^{r}(p, R)\,[G^{(+)}(p, R)-G^{(-)}(p, R)]\,\tilde{u}^{r}(p, R). \end{aligned} \tag{34}$$

Dictated by the commutation relation of the field operators, the spectral function obeys a sum rule, which reads

$$\int \frac{\mathrm{d}p_0}{2\pi} A(p, R)=\frac{M_p^*}{E_p^*}, \quad E_p^*=[(\boldsymbol{p}^*)^2+M_p^{*2}]^{1/2}. \tag{35}$$

[#3] The Poisson bracket denotes $\{A, B\}=\partial_P A\cdot\partial_R B-\partial_R A\cdot\partial_P B$.

[#4] We choose the normalisation of the effective spinors according to the covariant way [12], i.e. $\bar{\tilde{u}}^{r}(p, R)\,\tilde{u}^{s}(p, R)=\delta_{rs}$.

To find the spectral function, we use the second expression in the definition (34). We thus have to solve the equations of motion of $G^{(\pm)}$. Therefore we add eq. (23) and its adjoint, Wigner transform it and take the first-order approximation in the gradients. We obtain

$$\tfrac{1}{2}\mathrm{i}\partial_\mu^{\mathrm{R}}(\gamma^\mu G^{(\pm)}-G^{(\pm)}\gamma^\mu)+p_\mu(\gamma^\mu G^{(\pm)}+G^{(\pm)}\gamma^\mu)-2MG^{(\pm)}$$
$$=2+\Sigma^{(\pm)}G^{(\pm)}+G^{(\pm)}\Sigma^{(\pm)}+\tfrac{1}{2}\mathrm{i}(\{\Sigma^{(\pm)},G^{(\pm)}\}+\{G^{(\pm)},\Sigma^{(\pm)}\}). \tag{36}$$

Now we sandwich this equation between effective spinors and invert the resulting equation. The solution reads

$$\bar{u}^r G^{(\pm)}\tilde{u}^{r'}=\delta_{rr'}M_p^*[p_\mu p^{*\mu}-MM_p^*-\Sigma^{\mathrm{s}}M_p^*+(\Sigma^{\mathrm{v}})_\mu p^{*\mu}\pm\mathrm{i}\varepsilon]^{-1}. \tag{37}$$

With this result the spectral function A can be found to be

$$A(p,R)=\mathrm{i}M_p^*\left(\frac{2\mathrm{i}\,\mathrm{Im}(\Sigma^{\mathrm{s}}M_p^*-(\Sigma^{\mathrm{v}})_\mu p^{*\mu})}{[p_\mu p^{*\mu}-MM_p^*-\mathrm{Re}(\Sigma^{\mathrm{s}}M_p^*-(\Sigma^{\mathrm{v}})_\mu)]^2+[\mathrm{Im}(\Sigma^{\mathrm{s}}M_p^*-(\Sigma^{\mathrm{v}})_\mu p^{*\mu})]^2}\right). \tag{38}$$

Quite analogously to the non-relativistic treatment [2], we define the quasiparticle approximation appropriate if $\mathrm{Im}(\Sigma^{\mathrm{s}}M_p^*-(\Sigma^{\mathrm{v}})_\mu p^{*\mu})\ll\mathrm{Re}(\Sigma^{\mathrm{s}}M_p^*-(\Sigma^{\mathrm{v}})_\mu p^{*\mu})$. In that case we obtain for the spectral function

$$A(p,R)=2\pi M_p^*\delta(p^{*2}-M_p^{*2}). \tag{39}$$

Note that spectral function (39) satisfies our sum rule (35). Now that we obtained this expression for the spectral function, we return to the kinetic equation (24). Let us first deal with the LHS. or drift-term of it. From the related non-relativistic treatment [2], it is known that there is a one-to-one correspondence of the functional form of the spectral function (38) to the terms in the drift-term. In particular, the last term on the LHS of (24) corresponds to the spreading width in the spectral function. Since we neglected this width by taking the quasiparticle approximation, we consequently have to set this term zero in the kinetic equation. To write down the kinetic equation, we introduce two, to $F^{rr'}$ related functions:

$$\tilde{F}^{rr'}(p,R)=4\pi\theta(p_0^*)\delta(p^{*2}-M_p^{*2})F^{rr'}(p,R),$$
$$f^{rr'}(p,R)=\int\frac{\mathrm{d}p_0}{2\pi}E_p^*\tilde{F}^{rr'}(p,R). \tag{40}$$

The current J^μ defined as $J^\mu(x)=-\mathrm{i}(\gamma^\mu)_{\alpha\alpha'}G^<_{\alpha'\alpha}(x,x)$ is expressed in terms of f as

$$J^\mu(R)=\sum_r\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^{*\mu}}{E_p^*}f^{rr}(p,R), \tag{41}$$

from which we conclude that the scalar function f corresponds to the baryon density. By taking the Wigner-transform of (24), together with approximation (28), we obtain after some trivial algebra the exact result

$$p_\mu^*\partial_R^\mu\tilde{F}^{rr'}(p)-M_p^*\{\mathrm{Re}\,\Sigma^{\mathrm{s}}(p),\tilde{F}^{rr'}(p)\}+p_\mu^*\{\mathrm{Re}(\Sigma_{\mathrm{v}})^\mu(p),\tilde{F}^{rr'}(p)\}=-(\mathrm{RHS}). \tag{42}$$

Integration of (42) over p_0, assuming that there is no mixing of effective positive and negative energy states ($\theta(p_0^*)$ and $\theta(-p_0^*)$), gives the final equation [#5] (E_p denotes the single-particle energy $E_p=R_p^*-\mathrm{Re}\,\Sigma_0$)

$$(\partial_T+\nabla_pE_p\cdot\nabla_R-\nabla_RE_p\cdot\nabla_p)f_1(p)=\tfrac{1}{2}\operatorname*{tr}_{(2)}\int\frac{\mathrm{d}^3p_2\mathrm{d}^3p_1'\mathrm{d}^3p_2'}{(2\pi)^9}\frac{M_p^*M_{p_2}^*M_{p_1'}^*M_{p_2'}^*}{E_p^*E_{p_2}^*E_{p_1'}^*E_{p_2'}^*}$$
$$\times(\{\langle pp_2|T_{12}^{(+)}|p_1'p_2'\rangle f_1(p_1')f_2(p_2')\langle p_1'p_2'|T_{12}^{(-)}|pp_2\rangle_{\mathrm{A}}[1-f_1(p)][1-f_2(p_2)]+\mathrm{h.c.}\}$$
$$-\{\langle pp_2|T_{12}^{(+)}|p_1'p_2'\rangle[1-f_1(p_1')][1-f_2(p_2')]\langle p_1'p_2'|T_{12}^{(-)}|pp_2\rangle_{\mathrm{A}}f_1(p)f_2(p_2)+\mathrm{h.c.}]\})$$
$$\times(2\pi)^4\delta^3(\boldsymbol{p}+\boldsymbol{p}_2-\boldsymbol{p}_1'-\boldsymbol{p}_2')\,\delta(E_p+E_{p_2}-E_{p_1'}-E_{p_2'}). \tag{43}$$

[#5] The state $|p_1^{r_1}p_2^{r_2}\rangle_{\mathrm{A}}$ denotes the antisymmetrised state $|p_1^{r_1}p_2^{r_2}\rangle_{\mathrm{A}}=|p_1^{r_1}p_2^{r_2}\rangle-|p_2^{r_2}p_1^{r_1}\rangle$.

Here we used the fact that $\delta(p^{*2}-M_p^{*2})$ is a solution of the LHS of eq. (42). For the sake of clearness, we omitted the overall R dependence and denoted the kinetic equation (43) as an operator equation in spin space (indicated by the trace-operator). The appearing T-matrix elements are defined as

$$\begin{aligned} &p_1'^{r_1'} p_2'^{r_2'} | T^{(\pm)} | p_1^{r_1} p_2^{r_2} \rangle \cong \bar{u}^{r_1'}(p_1') \bar{u}^{r_2'}(p_2') \langle p_1' p_2' | T^{(\pm)} | p_1 p_2 \rangle \tilde{u}^{r_1}(p_1) \tilde{u}^{r_2}(p_2) \\ &\quad = \langle \boldsymbol{p}_1'^{r_1'} \boldsymbol{p}_2'^{r_2'} | T^{(\pm)}(E_{p_1}+E_{p_2}) | \boldsymbol{p}_1^{r_1} \boldsymbol{p}_2^{r_2} \rangle (2\pi)^3 \delta^3(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}_1'-\boldsymbol{p}_2'), \end{aligned} \tag{44}$$

where $\langle \boldsymbol{p}_1' \boldsymbol{p}_2' | T_{\frac{1}{2}}^{(\pm)}(z) | \boldsymbol{p}_1 \boldsymbol{p}_2 \rangle$ satisfies (again as operators in spin space)

$$\begin{aligned} &\langle \boldsymbol{p}_1' \boldsymbol{p}_2' | T_{\frac{1}{2}}^{(\pm)}(z) | \boldsymbol{p}_1 \boldsymbol{p}_2 \rangle = \langle \boldsymbol{p}_1' \boldsymbol{p}_2' | V_{12}(z) | \boldsymbol{p}_1 \boldsymbol{p}_2 \rangle \\ &\quad + \int \frac{\mathrm{d}^3 p''}{(2\pi)^3} \frac{M^*_{P/2+p''} M^*_{P/2-p''}}{E^*_{P/2+p''} E^*_{P/2-p''}} \langle \boldsymbol{p}_1' \boldsymbol{p}_2' | V_{12}(z) | \tfrac{1}{2}\boldsymbol{P}+\boldsymbol{p}'' \tfrac{1}{2}\boldsymbol{P}-\boldsymbol{p}'' \rangle \frac{1-f_1(\frac{1}{2}P+p'')-f_2(\frac{1}{2}P-p'')}{z-E_{P/2+p''}-E_{P/2-p''} \pm \mathrm{i}\varepsilon} \\ &\quad \times \langle \tfrac{1}{2}\boldsymbol{P}+\boldsymbol{p}'' \tfrac{1}{2}\boldsymbol{P}-\boldsymbol{p}'' | T_{\frac{1}{2}}^{(\pm)}(z) | \boldsymbol{p}_1 \boldsymbol{p}_2 \rangle . \end{aligned}$$

The kinetic equation (43) together with the effective interaction given by eq. (45) represent a self-consistent set of equations. Written in this way, eq. (43) resembles very much the VUU-equation. However, there is a difference in the collision term. The usual gain and loss contributions of it, now consists out of two terms each. These two terms only differ in their spin structure and consequently become identical when we consider a spin independent system. In that case the collision term becomes a relativistic generalisation of the Uehling–Uhlenbeck collision term. The drift term of the kinetic equation (43) corresponds to the one used in calculations by Ko et al. [13,14], Molitoris et al. [15] and the Giessen group [16]. In the theoretical studies of Vasak et al. [17,18] and Ivanov [19] also negative energy particles are considered. Their drift terms are consistent with our equation (42) where the integration over p_0 is not performed. However, both authors do not derive a collision term. As a last comparison we note that when neglecting all medium dependent effects and leave out anti-symmetrisation, we recover the kinetic equation of the Groot et al. [3].

The effective interaction given by eq. (45) in fact is a generalised Brueckner interaction. It is somewhat fancier than the conventional Brueckner interaction [11] for two reasons. Firstly, the blocking term in the propagator accounts for particle–particle and for hole–hole intermediate states. This can be seen by the equality $1-f_1-f_2=(1-f_1)(1-f_2)-f_1f_2$. The hole–hole part is neglected in the conventional Brueckner scheme. The second sophistication concerns the self-energies involved in the calculation of the single-particle energies E_p. From eq. (25) one sees that $\Sigma^{(\pm)}$ contains two terms. The first is the usual Brueckner mean-field term while the second accounts for contributions coming from collisions. For more details we refer to ref. [9].

This work is part of the research program of the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM) with financial support of the "Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek" (ZWO).

References

[1] P.C. Martin and J. Schwinger, Phys. Rev. 115 (1959) 1342.
[2] L.P. Kadanoff and G. Baym, Quantum statistical mechanics (Benjamin, New York, 1962).
[3] S.R. de Groot, W.A. van Leeuwen and Ch.G. van Weert, Relativistic kinetic theory (North-Holland, Amsterdam, 1980).
[4] W. Botermans and R. Malfliet, Phys. Lett. B 171 (1986) 22.
[5] W. Botermans and R. Malfliet, Proc. 8th Balaton Conf. on Nuclear physics (June 1987), ed. Z. Fodor, p. 86.
[6] W.D. Brown, R.D. Puff and L. Wilets, Phys. Rev. C 2 (1970) 332.
[7] L.V. Keldysh, Zh. Eksp. Teor. Fiz. 47 (1964) 1515 [Sov. Phys. JETP 20 (1965) 235].
[8] R.A. Craig, J. Math. Phys. 9 (1968) 605.
[9] W. Botermans and R. Malfliet, to be published.

[10] G. Baym, Phys. Rev. 127 (1962) 1391.
[11] B. ter Haar and R. Malfliet, Phys. Rep. 149 (1987) 207.
[12] C. Itzykson and J.-B. Zuber, Quantum field theory (McGraw-Hill, New York, 1980).
[13] C.M. Ko, Q. Li and R. Wang, Phys. Rev. Lett. 59 (1987) 1084.
[14] C.M. Ko and Q. Li, Phys. Rev. C 37 (1988) 2270.
[15] J.J. Molitoris et al., Phys. Rev. C 37 (1988) 1014.
[16] B. Blättel, V. Koch, W. Cassing and U. Mosel, Universität Giessen preprint UGI-88-6.
[17] D. Vasak, M. Gyulassy and H.Th. Elze, Ann. Phys. 173 (1987) 462.
[18] H.Th. Elze, M. Gyulassy, D. Vasak, H. Heinz, H. Stöcker and W. Greiner, Mod. Phys. Lett. A 2 (1987) 451.
[19] Yu.B. Ivanov, Nucl. Phys. A 474 (1987) 669.